12.3 Videos Guide

12.3a

- Definitions of the dot product: Let $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$
  - $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$
    - Exercise:

      Find the dot product $\mathbf{a} \cdot \mathbf{b}$ of $\mathbf{a} = \langle 2, -4, 7 \rangle$ and $\mathbf{b} = \langle -\frac{1}{2}, 5, 13 \rangle$.
  - $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$
    - Exercise:

      Find the angle between $\mathbf{a}$ and $\mathbf{b}$ from the previous exercise.
- Orthogonal vectors

  If the angle between $\mathbf{a}$ and $\mathbf{b}$ is $\theta = 90°$, then $\mathbf{a} \cdot \mathbf{b} = 0$

12.3b

- Direction cosines: let α, β, and γ be angles made between a vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and the x-, y-, and z- axes, respectively
  - $\cos \alpha = \frac{a_1}{|\mathbf{a}|}$
  - $\cos \beta = \frac{a_2}{|\mathbf{a}|}$
  - $\cos \gamma = \frac{a_3}{|\mathbf{a}|}$

12.3c

- Scalar projection of $\mathbf{b}$ onto $\mathbf{a}$
  - $\text{comp}_{\mathbf{a}} \mathbf{b} = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|}$
- Vector projection of $\mathbf{b}$ onto $\mathbf{a}$
  - $\text{proj}_{\mathbf{a}} \mathbf{b} = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|^2} \mathbf{a}$

12.3d

Exercise:

- Find the scalar and vector projections of $\mathbf{b}$ onto $\mathbf{a}$.

  $\mathbf{a} = \langle -1, 4, 8 \rangle$, $\mathbf{b} = \langle 12, 1, 2 \rangle$